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ALLOCATION OF INDIRECT COSTS

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ALLOCATION OF INDIRECT COSTS*

I. INTRODUCTION

STATEMENT OF THE PROBLEM

Manufacturing cost accounting procedures usually provide for relating all operating costs of the firm to the specific end products or services which the firm produces. For example, if a firm manufactures two products x and y, cost accounting would normally provide for accumulating the direct labor and material cost incurred in the production of x and y. In addition, it would also provide for distributing x and y's estimated pro-rata shares of manufacturing overhead (burden) and possibly general administrative and selling expenses to the end products, x and y.¹ Thus, in the final analysis all the operating costs of the firm, including the cost of "indirect" or "supporting" activities, are treated as being attributable to the end products.

A similar situation is sometimes found in the realm of government. For example, a certain governmental department or agency may engage in several end product (or service) activities; and for budgeting and accounting or other purposes the agency may wish to relate all its operating costs, including so-called "indirect" costs, to the end product activities.² This is particularly

* The author is very much indebted to H. Markowitz and R. T. Nichols of The RAND Corporation for helpful comments, criticisms, and suggestions.

1 J. J. W. Neuner, Cost Accounting Principles and Practices (Chicago: R. D. Irwin, Inc., 1942), pp. 3, 227.

2 In the case of economic planning for multiple purpose projects (e.g., projects like TVA), attempting to allocate all costs to the end product activities may not be a desirable or even correct procedure, particularly where true joint costs are involved. For a good discussion of this point see Joseph S. Ransmeir, The Tennessee Valley Authority: A Case Study in the Economics of Multiple Purpose Stream Planning (Vanderbilt University Press, 1942). The present paper is concerned only with those cases where cost allocation is deemed appropriate.

true if the governmental unit in question is operating in accordance with the principles of true performance budgeting.³

Stated very briefly, the present paper is concerned with a solution (not the only solution) to the cost distribution problem that arises when the operating costs of interdependent indirect ("support") activities are to be allocated to end product ("direct") activities.

A simple illustrative example. In order to pose the problem in an explicit manner, a simple illustrative example may be helpful. Consider a system which is composed of three support ("indirect") activities and two end product activities. Each of the three support activities furnishes services to every other activity in the system (including itself). The two end product activities for all practical purposes may be assumed independent of each other and may be assumed to furnish no services to the support activities. In other words the end product activities are true "end-item" operations and the support activities exist solely for the purpose of servicing the end product activities, although in the process of doing so they in fact support each other. This scheme of things may be diagrammed as shown in Figure 1, where activities 1, 2, and 3 are "support" and activities 4 and 5 are "end product."

³ Performance budgeting is discussed in the Hoover Commission report Budgeting and Accounting, A Report to the Congress, February 1949. The following passage from this report conveys the general idea of performance budgeting: "Such an approach would focus attention upon the general character and relative importance of the work to be done, or upon the service to be rendered, rather than upon the things to be acquired, such as personal services, supplies, equipment, and so on. These latter objects are, after all, only the means to an end. The all-important thing in budgeting is the work or the service to be accomplished, and what that work or service will cost." (Ibid., p. 6.)

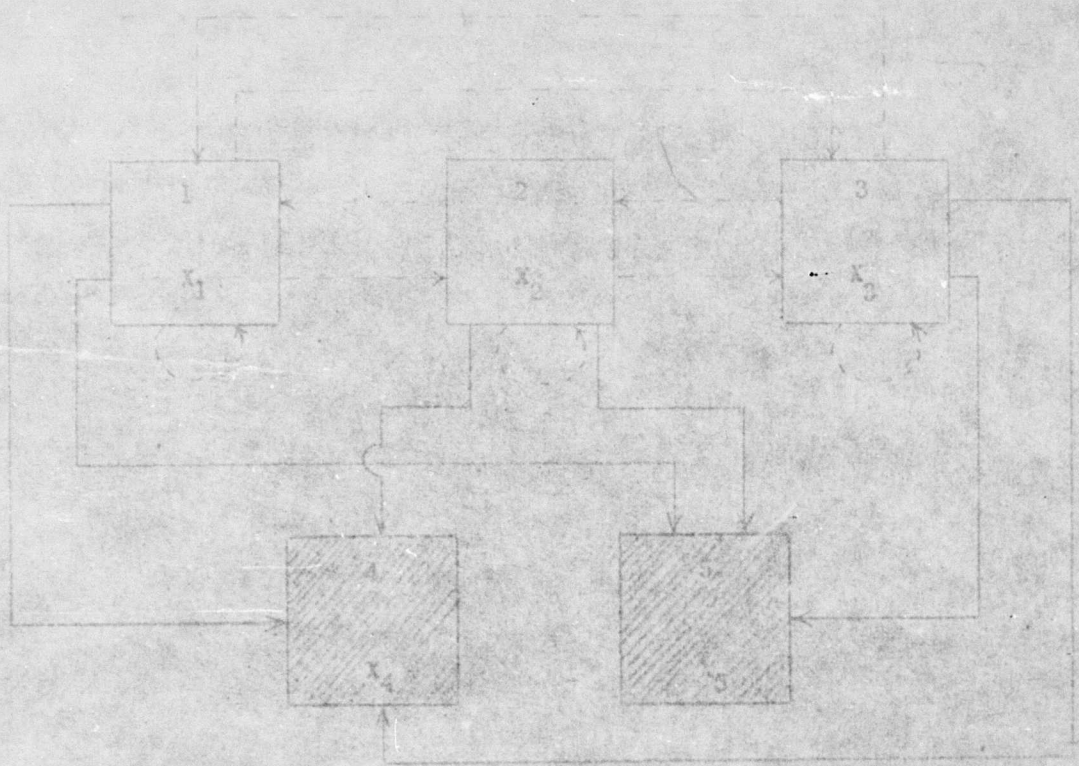


Figure 1

Under usual cost distribution procedures, the periodic operating costs of

The problem with which the present paper is concerned may now be stated very simply and explicitly. The problem is to allocate the operating costs X_1 , X_2 , X_3 to activities 4 and 5, taking into account inter-activity flows as illustrated in Figure 1.

METHOD OF SOLUTION

Apparently few present-day cost accountants would be so bold as to deny that most cost allocations are fairly arbitrary. In general they must be somewhat arbitrary because of the nature of the problem and also because the "cost of cost accounting" can rapidly become prohibitive. Carried to extreme, a firm could spend most of its time "pushing paper" and have very little time for production. Even then all of the arbitrariness could not be eliminated.

The cost-allocating procedure set forth in this paper is recognized as being somewhat arbitrary. Accordingly, it is proposed as a solution to the problem and not the solution. Potential users of the technique must consider carefully the assumptions built into the model. (These are discussed in the next section.) If the assumptions are excessively at variance with the conditions of the particular problem under consideration, then the model may be inappropriate. What is "excessive" in any specific case must, of course, be determined largely by the judgment of the person making the study.

The heart of the problem as formulated above stems from the condition of interdependence among the support activities. Therefore, any proposed solution must in some manner take this simultaneous interaction into account. The method set forth in this memorandum deals with the interdependence among support activities in one of the simplest possible ways: through the use of

a system of simultaneous linear equations.⁴

Only one thing need be known before the proposed cost allocation method can be used. For each support activity (1, 2, and 3 in Figure 1), some notion of the fraction of the support activity's total effort which went to each of the other activities during the accounting period under consideration must be made available. To make clear what is meant by this, let s_{ij} denote the fraction of the effort of activity j going to activity i during the period under consideration. Then for activity 1, for example,

s_{11} = fraction of activity 1's effort used to "support" itself

s_{21} = fraction of activity 1's effort which went to activity 2

.

s_{51} = fraction of activity 1's effort which went to activity 5,

where $s_{11} + s_{21} + s_{31} + s_{41} + s_{51} = 1$. Similarly, for support activities 2 and 3 we have respectively, $s_{12} + s_{22} + s_{32} + s_{42} + s_{52} = 1$, and $s_{13} + s_{23} + s_{33} + s_{43} + s_{53} = 1$.

The estimates of the s_{ij} can be as good or bad as local conditions and time permit in any particular application. Data from accounting and/or other records may be used as a basis for deriving the s_{ij} . Or, in some cases certain of the s_{ij} may have to be determined on the basis of judgment by the manager of the particular support activity in question.

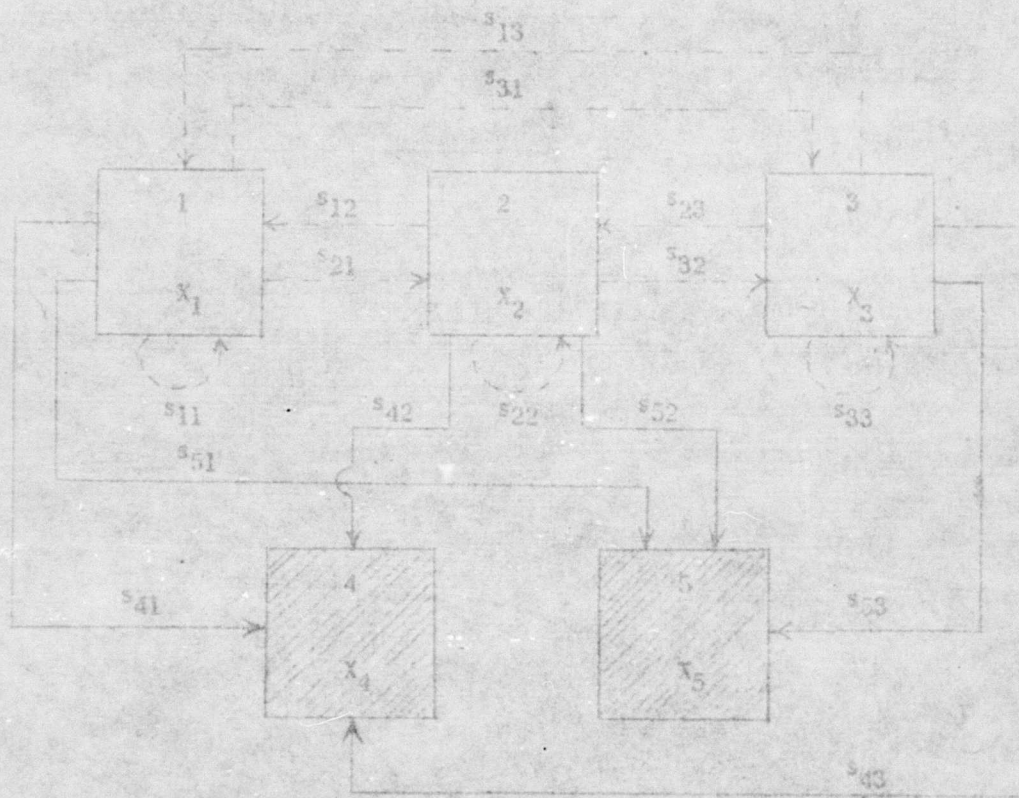
⁴ Representing the condition of interdependence in this manner is analogous to the "classical" accounting method of treating intercorporate stock holdings when deriving a consolidated balance sheet for several interdependent corporations. E.g., see Charles H. Tanager, Accounting Principles and Procedure (Advanced Accounting), Vol. II, Walton Publishing Company, Chicago, pp. 137-1 to 137-5.

ORGANIZATION OF REMAINDER OF THE PAPER

The proposed solution to the cost-allocation problem is treated in several stages. In the next section (Section II) the simple system portrayed in Figure 1 is discussed and an algebraic solution given. Next, in Section III the same problem is treated in the form of an illustrative numerical example and the solution presented, including a detailing of the computational steps required to obtain the solution. Finally, in Section IV the general case where there are m support activities and n end product activities is considered and an algebraic solution given.

II. SOLUTION OF THE COST ALLOCATION PROBLEM FOR A SIMPLE SYSTEM INVOLVING FIVE ACTIVITIES

In this section the simple system illustrated in Figure 1 of Section I is considered and an algebraic solution developed. For convenience of presentation the system is reproduced here as Figure 2 with the s_{ij} included in the diagram.



x_1 = cost (in dollars) of activity 1 for the accounting period under consideration.

s_{ij} = the fraction of the effort of activity j estimated to have gone to activity i during the period under consideration. ($\sum_i s_{ij} = 1$.)

Figure 2

As previously pointed out, the s_{ij} must be known beforehand; i.e., they are treated as "given data." Recalling that for fixed j the s_{ij} summed on i equals unity, we write down the following equations for the support activities:

$$(1) \quad 1 = \sum_{i=1}^5 s_{ij}, \quad (j = 1, 2, 3)$$

or rewriting,

$$\begin{aligned} (1-s_{11}) - s_{21} - s_{31} &= s_{41} + s_{51} \\ (2) \quad -s_{12} + (1-s_{22}) - s_{32} &= s_{42} + s_{52} \\ -s_{13} - s_{23} + (1-s_{33}) &= s_{43} + s_{53} \end{aligned}$$

Now suppose we introduce a number λ_{4j} , ($j = 1, 2, 3$), defined as the fraction of support activity j going to end product activity 4. We can then write down a system of equations to be solved for values of the λ_{4j} :

$$(3) \quad S \lambda_4 = s_4$$

where S is the matrix of s_{ij} on the left side of equations (2); λ_4 is a column vector with elements λ_{41} , λ_{42} , λ_{43} ; and s_4 is a column vector with elements s_{41} , s_{42} , and s_{43} . Solving (3) for λ_4 we get

$$(4) \quad \lambda_4 = S^{-1} s_4,$$

where S^{-1} is the inverse of the matrix S .

The solution (4) gives the fractions of the efforts of support activities 1, 2, and 3 going to end product activity 4, after taking into account the interdependence among the support activities. The major assumption involved here is that the interdependence among the support activities may be represented by the system of linear equations (3).

The fractions of the efforts of support activities 1, 2, and 3 devoted to end product activity 5 may be determined in the same manner as for activity 4. In this case we have

$$(5) \quad \lambda_5 = S^{-1}e_5.$$

Notice that S^{-1} in equation (5) is the same as in (4). Thus, the S matrix is the same in both cases, and only one matrix need be inverted. Furthermore, if in future accounting periods the s_{ij} are found to be essentially the same as in the current period, the same λ 's may be used to make cost allocations in future periods. In other words so long as the s_{ij} do not change, the λ 's may be used over and over again, thus making the computational task a very simple one.⁵

Having solved for λ_4 and λ_5 , and then taking the dollar costs (from the accounting records) for the various activities under consideration, we can compute the cost allocations to activities 4 and 5 as follows:

	<u>To Activity 4</u>	<u>To Activity 5</u>
$x_1 =$	$\lambda_{41} x_1 +$	$\lambda_{51} x_1$
(6) $x_2 =$	$\lambda_{42} x_2 +$	$\lambda_{52} x_2$
$x_3 =$	$\lambda_{43} x_3 +$	$\lambda_{53} x_3$
$\sum_{j=1}^3 x_j = \sum_{j=1}^3 \lambda_{4j} x_j + \sum_{j=1}^3 \lambda_{5j} x_j$		

⁵ There is also another possibility. If the s_{ij} ($i = 1, 2, 3$) do not change in future periods but the s_{ij} ($i = 4, 5$; $j = 1, 2, 3$) do change, then the λ 's will have to be recomputed. But this will not involve much effort because the S^{-1} matrix will not have changed. It is only when the s_{ij} ($i = 1, 2, 3$) change that one has to start all over again.

Equations (6) give the final results. The allocation of X_1 to the end product activities 4 and 5, after taking into account "interactivity flows" among the support activities, is given by the first equation in (6). The allocation of X_2 and X_3 is similarly given by the second and third equations respectively in (6); and the allocation of the total of the support activity costs to the end product activities is given by

$$\sum_{j=1}^3 \lambda_{4j} X_j \text{ and } \sum_{j=1}^3 \lambda_{5j} X_j.$$

The grand total of the costs of each of the end product activities is obtained as follows:

	<u>Activity 4</u>	<u>Activity 5</u>
Allocated support costs	$\sum_{j=1}^3 \lambda_{4j} X_j$	$\sum_{j=1}^3 \lambda_{5j} X_j$
"Direct" cost	X_4	X_5
Total	$X_4 + \sum_{j=1}^3 \lambda_{4j} X_j$	$X_5 + \sum_{j=1}^3 \lambda_{5j} X_j$

A numerical example of the technique is presented in the next section of the paper.

III. NUMERICAL EXAMPLE OF A SYSTEM INVOLVING FIVE ACTIVITIES

The purpose of this section is to give a numerical illustrative example of the system diagramed in Figure 2 of Section II. All computational steps required to obtain the final cost allocations are shown in chronological order.

Assume the following sets of (hypothetical) given data:

$X_1 = \$200$	$X_2 = \$900$	$X_3 = \$700$
$X_4 = \$300$	$X_5 = \$400$	
$s_{11} = .06$	$s_{12} = .15$	$s_{13} = .4$
$s_{21} = .3$	$s_{22} = .05$	$s_{23} = .26$
$s_{31} = .14$	$s_{32} = .4$	$s_{33} = .04$
$s_{41} = .3$	$s_{42} = .1$	$s_{43} = .2$
$s_{51} = \underline{.2}$	$s_{52} = \underline{.3}$	$s_{53} = \underline{.1}$
$\sum s_{i1} = 1.00$	$\sum s_{i2} = 1.00$	$\sum s_{i3} = 1.00$

Step 1. Compute the allocation of the individual activity costs (the X_i) on the basis of the s_{ij} :

From the given s_{ij} we form the S matrix:

$$\begin{vmatrix} .94 & -.30 & -.14 \\ -.15 & +.95 & -.40 \\ -.40 & -.26 & +.96 \end{vmatrix}$$

Then for λ_{4j} ($j = 1, 2, 3$), we have the following system of equations (corresponding to (3) in Section II):

$$\begin{aligned} .94\lambda_{41} - .30\lambda_{42} - .14\lambda_{43} &= .3 \\ -.15\lambda_{41} + .95\lambda_{42} - .40\lambda_{43} &= .1 \\ -.40\lambda_{41} - .26\lambda_{42} + .96\lambda_{43} &= .2 \end{aligned}$$

Solving this set of simultaneous equations for the λ 's, the results are:

$$\begin{aligned} \lambda_{41} &= .5338 \\ \lambda_{42} &= .4186 \\ \lambda_{43} &= .5441 \end{aligned}$$

Similarly for λ_{5j} , we have

$$\begin{aligned} .94\lambda_{51} - .30\lambda_{52} - .14\lambda_{53} &= .2 \\ -.15\lambda_{51} + .95\lambda_{52} - .40\lambda_{53} &= .3 \\ -.40\lambda_{51} - .26\lambda_{52} + .96\lambda_{53} &= .1, \end{aligned}$$

and solving for the λ 's, the results are:

$$\begin{aligned} \lambda_{51} &= .4662 \\ \lambda_{52} &= .5814 \\ \lambda_{53} &= .4559 \end{aligned}$$

Then to get the numerical values of the cost allocations for the given data assumed in this illustrative problem, we compute:

$$X_1 = \$200 = .5338 (200) + .4662 (200).$$

$$X_2 = 900 = .4186 (900) + .5814 (900)$$

$$X_3 = 700 = .5441 (700) + .4559 (700)$$

or,

$$X_1 = \$200 = \$106.76 + \$ 93.24$$

$$X_2 = 900 = 376.74 + 523.26$$

$$\underline{X_3 = 700 = 380.87 + 319.13}$$

$$\sum X = 1800 = 864.37 + 935.63$$

This gives the final allocations of the cost of the support activities to the end product activities, after taking into accounting the interdependence of the support activities.

To get the total cost of the end product activities, we take the allocations of support costs derived above and add the "direct" cost of the end product activities (X_4 and X_5):

	Activity No. 4	Activity No. 5	Total
Allocated "support" cost	\$ 864	\$ 936	\$1,800
"Direct" cost	300	400	700
Total	<u>\$1,164</u>	<u>\$1,336</u>	<u>\$2,500</u>

This final result gives the complete "slice" of operating cost for each of the end product activities.

IV. THE GENERALIZED CASE

In the preceding sections of this paper, only a very simple system was considered. This was done deliberately in order to simplify the exposition and to concentrate on the basic ideas of the cost allocation model.

Generalization of the technique is straightforward. In this section a brief outline of the generalized case is given.

For the general case we assume m support activities and n end product activities. Thus we have support activities 1, 2, 3, ..., m ; and end product activities $m + 1, m + 2, \dots, m + n$.

Using the same notation as that used in previous sections, the required given data are $X_1, X_2, \dots, X_m; X_{m+1}, \dots, X_{m+n}$; and

$$s_{ij}, \quad \begin{array}{l} i = 1, 2, \dots, m; m+1, \dots, m+n \\ j = 1, 2, \dots, m \end{array}$$

and for fixed j , $\sum_i s_{ij} = 1$.

The S matrix is the $m \times m$ square matrix:

$$\begin{vmatrix} (1 - s_{11}) & -s_{21} & \dots & -s_{m1} \\ -s_{12} & (1 - s_{22}) & \dots & -s_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ -s_{1m} & -s_{2m} & \dots & (1 - s_{mm}) \end{vmatrix}$$

We then introduce the number $\lambda_{m+k, j}$, defined as the fraction of support activity j going to end product activity $m + k$. To obtain the values of λ for the $(m+k)^{\text{th}}$ end product activity, we solve the system of equations

$$S \lambda_{m+k} = s_{m+k},$$

where S is the matrix defined above; λ_{m+k} is a column vector with elements $\lambda_{m+k, 1}, \lambda_{m+k, 2}, \dots, \lambda_{m+k, m}$; and s_{m+k} is a column vector with elements $s_{m+k, 1}, s_{m+k, 2}, \dots, s_{m+k, m}$. The solution is, of course,

$$\lambda_{m+k} = S^{-1} s_{m+k},$$

where S^{-1} denotes the inverse of the $m \times m$ matrix S .

The allocations of support costs to the end product activities $m+1, m+2, \dots, m+n$, may then be computed as follows:

$$\begin{array}{rcl} & \frac{m+1}{\lambda_{m+1, 1} X_1 +} & \frac{m+2}{\lambda_{m+2, 1} X_1 + \dots +} \quad \dots \quad \frac{m+n}{\lambda_{m+n, 1} X_1} \\ X_1 = & & \\ & \frac{m+1}{\lambda_{m+1, 2} X_2 +} & \frac{m+2}{\lambda_{m+2, 2} X_2 + \dots +} \quad \dots \quad \frac{m+n}{\lambda_{m+n, 2} X_2} \\ X_2 = & & \\ & \vdots & \vdots \quad \dots \quad \vdots \\ & \vdots & \vdots \quad \dots \quad \vdots \\ & \vdots & \vdots \quad \dots \quad \vdots \\ X_m = & \frac{m+1}{\lambda_{m+1, m} X_m +} & \frac{m+2}{\lambda_{m+2, m} X_m + \dots +} \quad \dots \quad \frac{m+n}{\lambda_{m+n, m} X_m} \end{array}$$

$$\sum_{j=1}^m X_j = \sum_{j=1}^m \lambda_{m+1, j} X_j + \sum_{j=1}^m \lambda_{m+2, j} X_j + \dots + \sum_{j=1}^m \lambda_{m+n, j} X_j$$

It is evident from the above presentation that the only computational problem of any consequence arises at the point of inverting the S matrix. This, however, is not apt to be very formidable in a practical application. Probably in most applications, the order of the S matrix would not exceed 15 or 20. Inversion of a 20×20 matrix is not a major problem for any reasonably well equipped IBM installation.

Furthermore, approximation devices may be used. For example the inverse of an $I-S$ matrix like the one given on page 14 may be approximated by the matrix $I+S$. Applying this method to the illustrative example of Section III,

the results are \$843 and \$957 as the support costs allocated to activities 4 and 5 respectively, as compared with the "exact" values \$864 and \$936.⁶

⁶ The author is indebted to Dr. Walter Jacobs of the Computation Division, Directorate of Management Analysis, DCS/C, HqUSAF for pointing out this approximation method and for making the computation for the numerical example.